Report

A Graphic Plot for a Soliton Solution of Sine-Gordon model of DNA

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ABSTRACT

There are many models of DNA, both the linear ones and the nonlinear ones. One interesting model in this regard is the sine-Gordon model of DNA as proposed by Salerno. It belongs to nonlinear model of DNA which is close to realistic model. Here we discuss a graphical plot of soliton solution of such a sine-Gordon model of DNA.

Key Words: soliton solution, sine-Gordon, DNA, graphic.

Introduction

There are many models of DNA, both the linear ones and the nonlinear ones [1]. One interesting model in this regard is the sine-Gordon model of DNA as proposed by Salerno [2], see also Daniel and Vasumathi [3]. It belongs to nonlinear model of DNA which is close to realistic model. A review of physical significance of such a sine-Gordon model was given in [6].

Here we discuss a graphical plot of soliton solution of such a sine-Gordon model of DNA.

Soliton solution of a sine-Gordon model of DNA

Assuming the wavefunction Ψ to be a function of x and t, then the sine-Gordon model of DNA can be written as follows: [3, p.7]

$$\Psi_{tt} - \Psi_{zz} + \sin(\Psi) = 0 \tag{1}$$

or in Mathematica expression:

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Ψ=U[x-c t];
pde=D[Ψ,x,x]-D[Ψ,t,t]-sin[Ψ]?0
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Now we will use Mathematica 9.0 to simplify and give graphical plot [3, p.443].To simplify with Mathematica:

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$$-\sin[U[z]] + U''[z] - c^2 U''[z] = 0 \tag{2}$$

The result is known as kink soliton wave: [3, p.444]

$$\Phi = 4\text{ArcTan}[c\text{Sinh}[x/\text{Sqrt}[1-c\text{^}2]]/\text{Cosh}[ct/\text{Sqrt}[1-c\text{^}2]]] \tag{3}$$

or in Mathematica:

$$4\text{ArcTan}\left[c\text{Sech}\left[\frac{ct}{\sqrt{1-c^2}}\right]\text{Sinh}\left[\frac{x}{\sqrt{1-c^2}}\right]\right]$$

Differentiating for t, it yields:

$$\partial_t\left(4\text{ArcTan}\left[c\text{Sech}\left[\frac{ct}{\sqrt{1-c^2}}\right]\text{Sinh}\left[\frac{x}{\sqrt{1-c^2}}\right]\right]\right)$$

$$-\frac{4c^2\text{Sech}[\frac{ct}{\sqrt{1-c^2}}]\text{Sinh}[\frac{x}{\sqrt{1-c^2}}]\text{Tanh}[\frac{ct}{\sqrt{1-c^2}}]}{\sqrt{1-c^2}(1+c^2\text{Sech}[\frac{ct}{\sqrt{1-c^2}}]^2\text{Sinh}[\frac{x}{\sqrt{1-c^2}}]^2)}$$

Simplifying the above result, it yields:

$$\text{Simplify}\left[-\frac{4c^2\text{Sech}\left[\frac{ct}{\sqrt{1-c^2}}\right]\text{Sinh}\left[\frac{x}{\sqrt{1-c^2}}\right]\text{Tanh}\left[\frac{ct}{\sqrt{1-c^2}}\right]}{\sqrt{1-c^2}\left(1+c^2\text{Sech}\left[\frac{ct}{\sqrt{1-c^2}}\right]^2\text{Sinh}\left[\frac{x}{\sqrt{1-c^2}}\right]^2\right)}\right]$$

$$-\frac{8c^2\text{Sinh}[\frac{ct}{\sqrt{1-c^2}}]\text{Sinh}[\frac{x}{\sqrt{1-c^2}}]}{\sqrt{1-c^2}(1-c^2+\text{Cosh}[\frac{2ct}{\sqrt{1-c^2}}]+c^2\text{Cosh}[\frac{2x}{\sqrt{1-c^2}}])}$$

The 3D plot is given below for c= 0.72

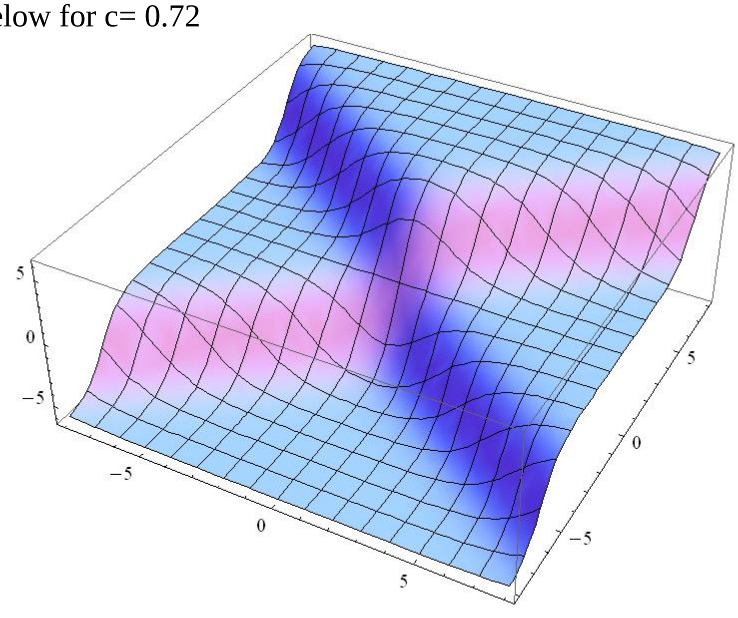

Figure 1. Mathematica plot of soliton solution on sine-Gordon equation for c=0.72

Perturbed Sine-Gordon Equation (SGE)

Perturbed SGE come in a variety of forms. One common form is a damped and driven SGE [7, p.17]:

$$\Psi_{tt} + \Phi\Psi_t - \Psi_{zz} + \sin(\Psi) = F \tag{4}$$

In addition, the following two versions of the perturbed SGE have been studied in the literature, including:

a. Directly forced SGE: [7, p.19]

$$\Psi_{tt} - \Psi_{zz} + \sin(\Psi) = Mf(\omega t) \tag{5}$$

b. Damped and drived SGE:

$$\Psi_{tt} - \Psi_{zz} + \sin(\Psi) = Mf(\omega t) - \alpha\Psi_t + \eta \tag{6}$$

In the meantime, (2+1)D SGE with additional spatial coordinate (y) is defined as [7,p.21]:

$$\Psi_{tt} = \Psi_{xx} + \Psi_{yy} - \sin(\Psi) \tag{7}$$

In their in-depth review of SGE, Ivancevic and Ivancevic [7] discuss potential applications of SGE solitons in DNA, protein folding, microtubules, neural impulse conduction and muscular contraction soliton. New insights may be expected in the near future in these biological fields, based on sine-Gordon equation soliton.

Conclusion

There are many models of DNA, both the linear ones and the nonlinear ones [1]. One interesting model in this regard is the sine-Gordon model of DNA as proposed by Salerno [2]. It belongs to nonlinear model of DNA which is close to realistic model. Here we have discussed a graphical plot of soliton solution of such a sine-Gordon model of DNA.

Considering that sine-Gordon equation has been used extensively by particle physicists, it would be interesting to study possibility to improve or alter DNA using electromagnetic field/pulse such as laser. This may be considered as a DNA enhancement method. New insights may be expected in the near future in these biological fields, based on sine-Gordon equation soliton.

References

[1] LudmilaV. Yakushevich.*Nonlinear Physics of DNA*. Second, rev. ed. Berlin: Wiley-VCH Verlag GmBH & Co., 2004.
[2] M. Salerno, *Phys. Rev*. A 44 (1991) 5292
[3] M. Daniel & V. Vasumathi. *Soliton-like base pair opening in a helicoidal DNA: An analogy with helimagnet and cholesterics*. arXiv:0812.4536 [nlin.PS], 2008.
[4] Richard H. Enns & George C. McGuire.*Nonlinear Physics with Mathematica for Scientists and Engineers*. Berlin: Birkhäuser, 2001, p. 443-445.
[5] Sadri Hassani. *Mathematical Methods using Mathematica: For Students of Physics and Related Fields*. New York: Springer-Verlag New York, Inc., 2003.
[6] Sara Cuenda, Angel Sanchez, & Niurka R. Quintero. Does the dynamics of sine-Gordon solitons predict active regions of DNA? *Physica D* 223 (2006) 214-221.
[7] Vladimir G. Ivancevic and Tijana T. Ivancevic. *Sine-Gordon solitons, Kinks and Breathers as Physical Models of Nonlinear Excitations in Living Cellular Structures*. arXiv:1305.0613 [q-bio.OT]