Article

Fibonacci Sunflowers, Mathematical Objects & Cubic Electrons

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Abstract

We examine the uncanny involvement and influence of mathematical truths on the minutest and largest properties and phenomena of the physical world and the remarkable effectiveness of mathematics in describing and predicting natural phenomena. Indeed, mathematics - a vast and ingenious conceptual structure without empirical content - is an indispensable and powerful theoretical instrument for the scientific understanding of virtually every aspect of the physical world. Selected examples show the remarkable associations among specific mathematical objects and macroscopic and quantum matter and phenomena.

Keywords: Mathematical object, Euler's identity Fibonacci sequence, golden ratio, wave-particle, duality, heavy electrons, Higgs boson.

Introduction

Space and time have been assigned aspects of empirical reality by geometers who invented the language and algebra of dimensionality. Prior to that, space and time were indifferently perceived as amorphous realities, devoid of metrics. Initially a *tabulae rasae*, with metrics, space and time became mathematical objects, e.g., geometric manifolds with measurable attributes: shapes, distances, topology, intervals. Euclid and other ancient geometers gave spatial dimensions axiomatic meaning by positing mathematical objects such as lines, triangles, circles, spheres, cubes and other regular and irregular geometric solids. Descartes recognized relations among geometric points and temporal instances, giving them meaning by means of algebraical coordinates and equations. This was followed by the notion of multi-dimensional spaces. Then, theoretical physicists elaborated on the geometric details by imbuing space and time with material meanings and properties. Accordingly, synthetical spaces ostensibly produce quantum particles, quantum waves and force fields that interact energetically, stochastically, and periodically in synthetical time.

Euclidian geometric space and various non-Euclidian abstract spaces of Gauss, Lobachevsky, Bolyai, and Riemann maintained the notion of three 'obvious' spatial dimensions. Then, along came the notion of four-dimensional 'spacetime' - the union of the three dimensions of space with the single dimension of time to create a combined entity. The 'deformation' of synthetical spacetime ostensibly explains gravitation. The notion of spacetime was introduced in the twentieth century by Hermann Minkowski and elaborated upon by Albert Einstein in his general theory of relativity. Current theoretical concepts of spacetime are not restricted to just four dimensions. In an attempt to explain quantum particles and waves, 'string' theory posits a ten-dimensional hyper-spacetime. 'M-theory', which is an elaboration of string theory, posits eleven-dimensional

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hyper-spacetime and mathematical objects that reside in it.

Calabi-Yau manifolds constitute an interesting class of synthetical geometric spaces, that is, mathematical objects which Shing-Tung Yau showed to be possible upon proving the Calabi conjecture [1]. These spaces or shapes are based on complex space-time coordinates which bifurcate into even numbers of ‘real’ and ‘complex’ dimensions. The six-dimensional case is of special interest to string theory, where it serves as a candidate for the geometry of the theory's six hidden, or extra 'compactified' dimensions. They are presumed to be compactified because they are not observed - they are infinitesimal in relation to the metrics of three dimensional space. This compactification notion is really an unnecessary ploy. After all, if we can assign four pseudo dimensions to spacetime, then we can keep adding more dimensions as needed to satisfy, perhaps never quench, the thirst analytical geometers and theoreticians have for even more multidimensional spaces.

Geometric compactification - the 'rolling up of a space' so that it is minute yet definable - is considered necessary for a viable string theory. In string theory different ways of compactifying, the extra dimensions lead to the notion that their modulations comprise elementary ‘physical objects’ such as electrons and photons. In this sense, synthetical spacetime ‘manifolds’ are considered to be *able* to manifest as physical objects. Various ‘vibrational modes’ of these spacetime manifolds have been taken as being the origins and constituents of quantum particles, fields and forces that pervade the cosmos. Thus, spacetime *origami* has emerged as a tool to fold-forge quantum particles.

Shin-Tung Yau in *The Shape of Inner Space* holds that the study of Calabi-Yau manifolds has provided a promising laboratory for thought experiments - a laboratory that can inform the physics of string theory and also cosmology. It is a testament to the agility of the human mind that we began thinking about Calabi-Yau's manifolds strictly as objects, albeit mathematical objects, before there was any obvious role for them in physics. Yau writes, “We're not forcing Calabi-Yau's on nature, but nature seems to be forcing them upon us.” The study of these compactified multi-dimensional manifolds has enabled both physicists and mathematicians to learn many interesting and unexpected things about the likely nature of elementary particles. Although Calabi-Yau spaces may not be the ultimate destination, they may well be important stepping-stones to the next level of understanding. Yau’s comments echo what Eugene Wigner called "The unreasonable effectiveness of mathematics in the natural sciences." Yau and Wigner marvel that laws of physics closely complement the mathematician's penchant for what is ‘elegant and beautiful’. At the same time, they grapple with the deep mystery of why this should be the case.

Mathematical Beauty & Truth

Euler's identity is often cited as an example of deep mathematical beauty. It is also a precise statement of mathematical truth which links three basic arithmetic operations: addition, multiplication, and exponentiation and also links five fundamental mathematical constants:

The number **0**, the null identity,

The number **1**, the unit identity,
The number $\boldsymbol{\pi}$, which is ubiquitous in the geometry of Euclidean space,
The number **e**, the base of natural logarithms which occurs widely in mathematical analysis,
The number $\boldsymbol{i}$, the imaginary unit of the complex field of numbers.
Both $\boldsymbol{\pi}$ and $\boldsymbol{e}$ are transcendental numbers.

Euler's identity (named after the Swiss mathematician Leonhard Euler) is the equality

$$e^{i\pi} + 1 = 0$$

e is Euler's number, the base of natural logarithms: **2.718281828** . . .
i is the imaginary unit, which satisfies $i^2 = -1$, and
π is the ratio of the circumference of a circle to its diameter: **3.14159265** . . .

Euler's identity is a special case from complex analysis, which states that for any real number **x**,

$$e^{ix} = \cos x + i \sin x$$

where the inputs of the trigonometric functions sine and cosine are given in radians. When $x = \pi$ radians, or *one half-turn (180°) around a circle*:

$$e^{i\pi} = \cos \pi + i \sin \pi$$

Since, $\cos \pi = -1$ and $\sin \pi = 0$ it follows that $e^{i\pi} = -1 + 0i$ which yields Euler's identity: $e^{i\pi} + 1 = 0$.

It should not be surprising that Euler's identity precisely describes, seemingly unrelated physical phenomena at macroscopic, microscopic, and quantum levels. Figure 1 illustrates how the Euler's identity is related to the -180^o flip of a playing card, which also corresponds to the act of turning a page, opening a book, a stenographer's tablet, or a laptop computer.

At the quantum level, Euler's identity describes the behavior of entangled quantum particles. Measurements of physical properties, such as spin, performed on entangled particles are found to be correlated by Euler's identity. For example, if a pair of particles is simultaneously generated in such a way that their total spin is zero, and one particle has a spin-vector 'up', then the spin-vector of the other particle is 'down', or a -180^o flip measured on the same coordinates axes. The entanglement paradox - although described by Euler's identity - is unexplained. It appears that one particle of an entangled pair 'knows' the spin measured on the other, even though there is no known means for such information to be communicated between the particles, which at the time of measurement may be separated by arbitrarily large distances [2].

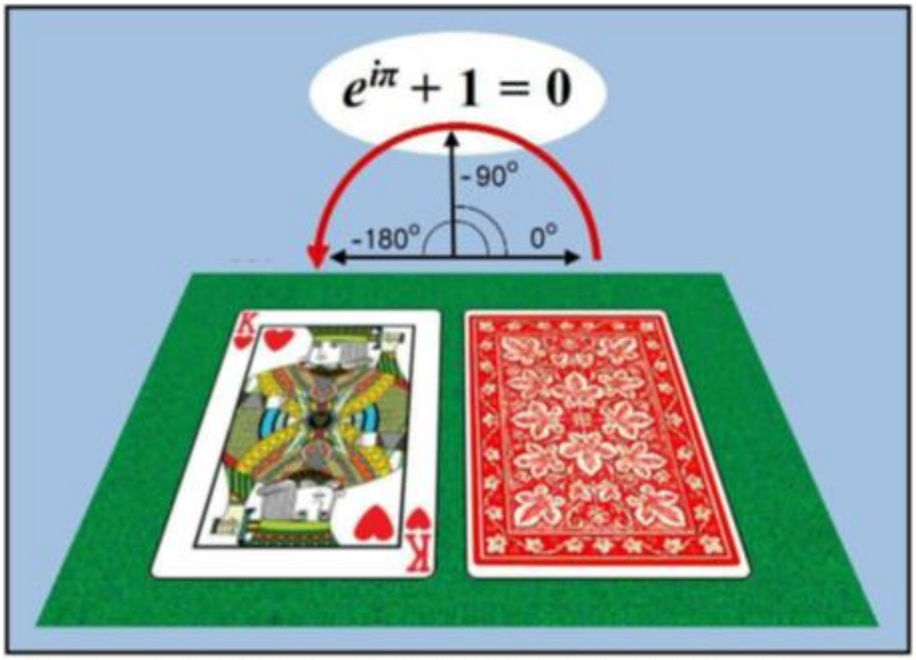

Figure 1 - Euler's identity and minus 180 degree flip of playing card.

Fibonacci Sequence & Golden Ratio

Fibonacci numbers appear unexpectedly and often in mathematics, so that there is an entire journal dedicated to their study. Applications of Fibonacci numbers include computer algorithms such as the Fibonacci search technique and the Fibonacci heap data structure. Fibonacci numbers also appear in biological settings such as branching in trees, the arrangement of leaves on a stem, the fruit sprouts of a pineapple, the arrangement of a pine cone's bracts, and prominently in sunflower seed patterns. The incremental number of petals on flowers often follows the Fibonacci sequence . . . 3, 5, 8, etc. - so that four-leaf clovers are extremely rare. Ratios of successive pairs of numbers in the Fibonacci sequence converge to the 'golden ratio' which appears in logarithmic spiral patterns in nature, including the spiral arrangement of leaves and other plant parts, snail and nautilus shells, and the cochlea of the inner ear. The ancient Greeks and modern architects and artists have felt compelled to apply the golden ratio in their structures and works to achieve elegance and beauty.

The Fibonacci numbers or Fibonacci sequence are the numbers in the following integer sequence:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, . . .

or the alternative sequence:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, . . .

By definition, the first two numbers in the Fibonacci sequence are either 1 and 1, or 0 and 1, depending on the chosen starting point of the sequence, and each subsequent number is the sum of the previous two

The Fibonacci spiral - an approximation of the golden spiral - is a logarithmic spiral whose growth factor is **φ**, the golden ratio. That is, a golden spiral gets wider (or further from its origin) by a factor of **φ** for every quarter turn it makes. Expressed algebraically, for two quantities **(a)** and **(b)** with **(a)** > **(b)** > **0**,

If $\mathbf{(a + b)/(a) = (a)/(b)}$, then, by definition $\mathbf{(a)/(b) = \varphi = (1+\sqrt{5})/(2) = 1.618033988}$ . . .

Simplification and substitution of $\mathbf{(b)/(a) = 1/\varphi}$ yields the relation: $\mathbf{\varphi^2 = \varphi + 1 = 2.618033988}$ . . .

The sequence of the ratios of successive pairs of Fibonacci numbers $\mathbf{F_{n+1}/F_n}$ converges to **φ**. Similarly, the sequence of the ratios of the sums of the first **n** Fibonacci numbers $\mathbf{\Sigma F_n / F_n}$ converges to $\mathbf{\varphi^2}$. The latter relationship is illustrated in Figure 2 and is ostensibly significant in delimiting the size of sunflowers, daisies, and other plants and perhaps even 'arms' of spiral galaxies.

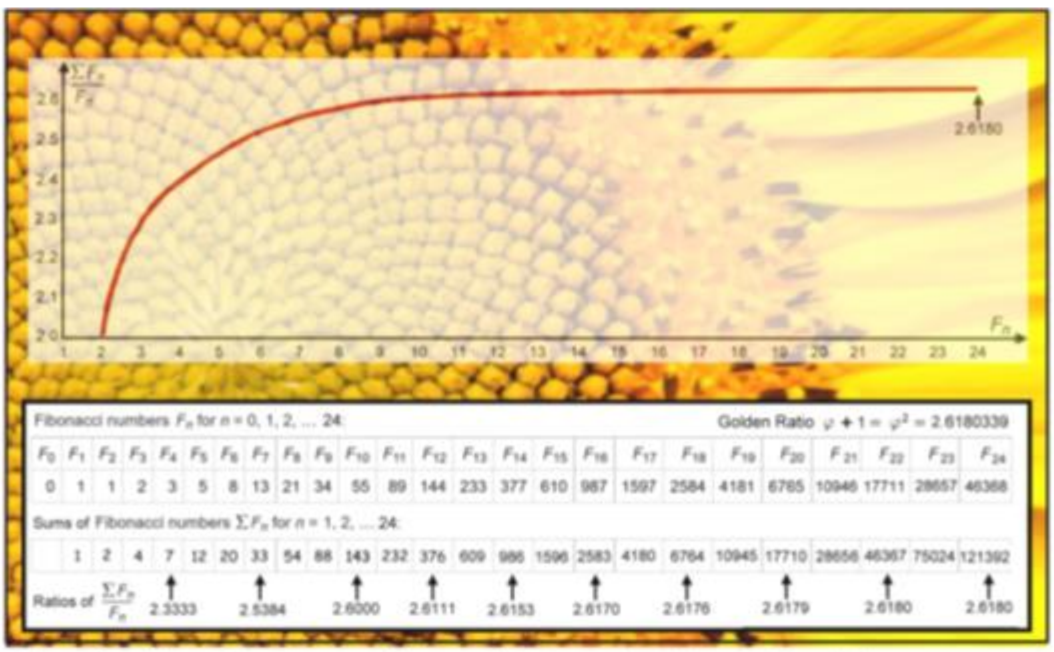

Figure 2 - Fibonacci sequence, sunflower seed patterns, and golden ratio interrelation.

The seed number increments of a sunflower follow the Fibonacci sequence as they fill the outside space away from the center. During the process, sunflower seeds form spiraling patterns. In some cases, the seed heads are so tightly packed that total number can get quite high - as many as 144 or a higher Fibonacci number. The total number of seeds tends to match a Fibonacci number. But, since the Fibonacci sequence never ends, what limits the total number of seeds in a sunflower? Why are there no sunflowers as big as dinner plates or tractor wheels? The obvious answer is that plant physiology controls growth factors which limit the diameter and size peculiar to sunflowers, daisies, and other flora - allowing the best exposure to sunlight and other factors. Yet, we observe from the curve in Figure 2 that beyond approximately Fibonacci number 144 the golden ratio $\mathbf{\varphi^2}$ is very quickly reached. We cautiously infer that the sunflower seed pattern spiral shape and size are governed and limited by the golden ratio $\boldsymbol{\varphi^2 = \varphi + 1 = 2.618033988}$.

The 'arms' of spiral galaxies tend to follow the Fibonacci pattern and golden ratio. The Milky Way and neighboring Andromeda galaxy have several spiral arms, each of them a logarithmic spiral. Like sunflowers, spiral galaxies appear to conform to size limitations which may be attributed to a range in mass content consistent with the golden ratio. Factors governing spiral galaxy size range apparently include so-called dark matter and dark energy and also the 'winding' constraints that typify the shape and extent of the spiral arms.

Wave-Particle Duality Myth

In the double-slit experiment light passing through two precisely cut parallel slits in a thin opaque plate is collected on a photo-sensitive detector screen. After the photons pass through the slits, the image produced on the detector screen consists of fringes, an interference pattern, with light and dark regions corresponding to where light waves have constructively and destructively interfered, Figure 3.

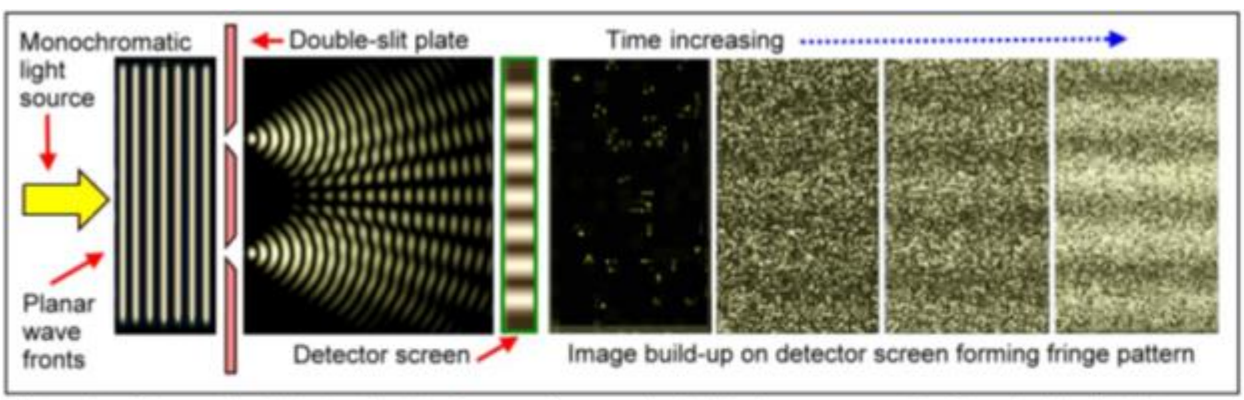

Figure 3 - Consructive & destructive interference of assumed cylindrical waves emanating from double-slit plate.

The interference pattern is taken as evidence of wave-particle duality, as inferred from the application of wave-function calculations. When photons or other quantum 'particles' pass through the double slit, a calculation which *assumes* that the photons behave as waves *is needed* - to predict the interference pattern. When the fringe pattern is examined, it is evident that the pattern was built piecemeal - quantum by quantum - not by continuous wave fronts - forming fringes composed of a patterned accumulation of random spots.

During transit, each photon, from the source to detector screen through one or the other double slit evolves, according to the Schrödinger wave-function, and spreads out in space as an assumed wave front. But the actual measurement on the detector screen finds each photon deposited as a quantum parcel at a particular spot on the detector surface. This is unanticipated by the wave-function - it is called wave-function collapse. This is explained by assuming wave-particle duality, that is, photons in transit are waves which become particles as they impinge the detector screen. It is better to admit that photons are *photons* and to avoid describing *photons* as either waves or particles.

Interfering cylindrical wave fronts are mathematical objects which are *assumed* to describe photons in transit because they effectively predict the *probable* fringe pattern of illumination and photo-excitation of *actual* particles that comprise the detector screen photo-sensitive granules. The mystery still remains in that the pattern is built piecemeal - quantum by quantum - not by the diffuse spreading of continuous wave fronts.

An alternative way to describe the transit of photons - or other quantum 'particles' - is based on a Louis de Broglie concept adopted and developed by David Bohm. He postulated that photons travel through both slits, but that each photon has a well-defined trajectory and passes through exactly one of the slits. According to Bohm, photon interactions govern the trajectory of each photon instant by instant. This implies a definable mathematical and empirical 'configuration space' which guides the photons to their destinations [3]. The de Broglie - Bohm 'pilot-wave' theory attempts to explain the behavior of photons without reference to wave-particle duality - the associated particle is superfluous since pure wave theory is deemed sufficient.

Super-Heavy Cubic Electrons

In 1936, Einstein expressed the counter-intuitive notion that "perhaps we must give up, by principle, the spacetime continuum." Current quantum gravity models assume that spacetime is discontinuous and particulate - that spacetime is a fractal *substance* and that the granularity of spacetime is on the scale of Planck length (10^{-33} centimeter) and Planck interval (10^{-43} second) [4]. In agreement, we postulate that spacetime consists of energetic 'four-dimensional' parcels - dynamic mathematical objects which consist of cubical volumes (voxels) of space that oscillate in time - and that spacetime is a deformable substance consisting of tightly tesselated voxels. This corresponds to Einstein's concept of a deformable spacetime, that is, a substantive material-like spacetime. The tensor calculus of his general theory of relativity demanded it. According to the theory, gravitational deformation of spacetime adjacent to the sun is analogous to the deformation of a trampoline surface by a heavy bowling ball tossed upon it.

Modified spacetime voxels (cubic spacetime units) are also proposed as fundamental building blocks of subatomic quantum parcels such as neutrinos, electrons, positrons. It will be seen that this leads to the prediction of heavy electron masses which agree with and surpass the Standard Model of Particle Physics.

Dimensional analysis of the equation, $\mathbf{E = mc^2}$ indicates a profound relation between the ratio $\mathbf{(E/m)}$ and the space/time ratio $\mathbf{[L/T]^2}$. The explicit meaning of the equation is that nuclear binding energies and mass defects are related - that matter (mass) and energy are interchangeable and complementary. The implicit meaning of $\mathbf{E = mc^2}$ is that energy and mass are essentially properties of space and time, that is, space-displacement $\mathbf{[\Delta L]^2}$ and time-interval $\mathbf{[\Delta T]^2}$. We postulate that select spacetime parcels (voxels) have energy and mass and contribute energy and mass - when combined as components of sub-atomic quantum parcels. This idea is applied in an abstract cubic representation of electrons according to the following *thought experiment* which models electrons as cubelike mathematical objects [5].

The basic electron, as depicted in Figure 4, is essentially naked and may enjoy only a transitory existence. The naked electron is represented by the intersection of three mutually orthogonal strings and branes which carry mass, spin, and charge (a borrowing from string and M-theory). These properties are purely energetic potentialities and at this stage are undefined as physically measurable properties. According to our cubic electron model, after the naked electron is cloaked with a cubic spacetime voxel, its electric charge, spin, and mass appear as measurable properties. It then becomes a **Generation 1** electron with measurable mass of **0.511 MeV** and also conforms with Standard Model electric charge (-1) and spin (½) properties.

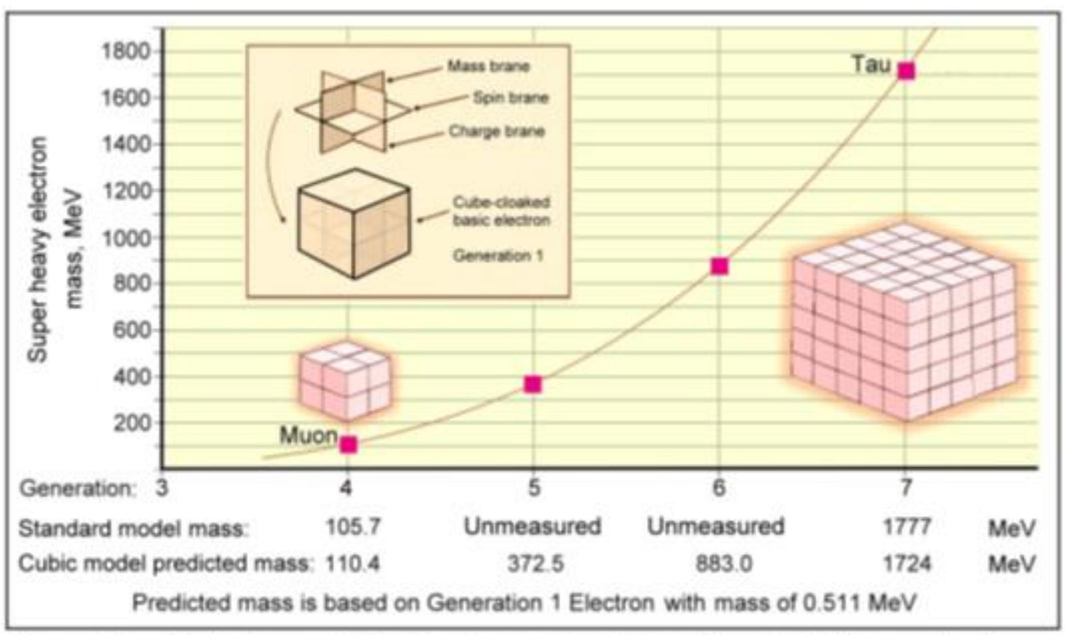

Figure 4 - Cubic heavy electrons compared with standard model heavy electrons.

A **Generation 2** electron consists of 8 **Generation 1** modules and $2^3 \bullet 0.511 = 4.088$ MeV.
A **Generation 3** electron consists of 27 **Generation 1** modules and $3^3 \bullet 0.511 = 13.797$ MeV.
The charge (-1) and spin (½) remain the same for all generations of heavy electrons.

We use the **Generation 3** cubic electron module for generating super heavy electrons (Figure 4).
A **Generation 4** electron consists of 8 **Generation 3** modules and $2^3 \bullet 13.797 = 110.4$ MeV.
A **Generation 5** electron consists of 27 **Generation 3** modules and $3^3 \bullet 13.797 = 372.5$ MeV.
A **Generation 6** electron consists of 64 **Generation 3** modules and $4^3 \bullet 13.797 = 883.0$ MeV.
A **Generation 7** electron consists of 125 **Generation 3** modules and $5^3 \bullet 13.797 = 1724$ MeV.

Fermat's Last Theorem seems to govern the formation of cubic electrons. For example, a **Generation 4** and a **Generation 5** cubic electron cannot be added together to form a more massive cubic electron - because, according to the theorem, the sums of cubes of successive integers are not closed under addition. Fermat's Last Theorem states that for **n > 2** no integers **A, B, C, n** can be found such that $\mathbf{A^n + B^n = C^n}$.

The cubic electron model not only predicts the muon and tau (tauon) masses but shows seven generations (with more possible) where the Standard Model projects only two generations of heavy electrons. The slight discrepancies between the Standard Model and cubic heavy electron mass predictions may be due to measurement anomalies with these highly transient particles. The Standard Model holds that there are three generations of electrons: The first generation being the basic electron itself, while the second generation is the heavier muon, and the third, the even heavier tauon. All three possess the same charge (-1) and spin (½) as an electron but differ greatly in mass.

Muons were discovered by Carl D. Anderson and Seth Neddermeyer at Caltech in 1936, while studying cosmic radiation. The muon is a theoretically predicted heavy electron with a mass of 105.7 MeV. The tau was detected in a series of experiments between 1974 and 1977 by Martin Lewis Perl with his colleagues at the SLAC National Accelerator Laboratory and the Lawrence Berkeley Laboratory group. The tauon mass is adduced as 1777 MeV. In their free state, these heavy electrons decay almost instantly.

Some types of heavy electrons are found within a number of lanthanide and actinide compounds, where they exhibit a large effective mass, comparable to the mass of a muon. Heavy electron mass may reach 1,000 MeV in exotic ‘heavy fermion’ materials. A microscope that was designed to image electron arrangements and interactions in crystals revealed electrons that possess extraordinary mass under certain extreme conditions. These heavy electrons appear to be confined electrons as they interact with crystal lattices that they traverse. Apparently, under extreme conditions, unusual phase transitions occur in particular materials causing electrons to take on extraordinary mass. These heavy electrons are apparently unstable unless they interact with and acquire mass in certain crystalline lattice structures.

This thought experiment adopts the idea that spacetime is particulate and substantive - and that certain manifestations spacetime voxels appear as electrons. The Standard Model of Particle Physics postulates the existence of the Higgs boson which imparts to electrons and other elementary particles gravitational mass and inertia. The Higgs boson was inserted in the Standard Model ad hoc - along with other elementary particles which are grouped in an arbitrary way and which depend on 24 numbers whose values cannot be deduced from first principles, but which need to be chosen to fit the observations. Although the Standard Model is imperfect, it is considered adequate for present purposes. Spacetime voxels, as conceived here, have the properties attributed to the Higgs boson and may incrementally impart mass to elementary particles.

Conclusion

The previous discussion elaborates on the uncanny relations among pure, perfect mathematical objects and physical, material, measurable, objects to the quantum level. In the case of Euler's identity we note that even one of the most esoteric mathematical statements relates to mundane objects and actions. The Fibonacci sequence and golden ratio are so pervasive that one is tempted to conclude that objective reality is governed by the sequence and ratio. Of course, they are simply mathematical descriptors which have observable manifestations. Other mathematical constructs, such as the de Broglie - Bohm 'pilot-wave' theory describe unobservable quantum phenomena and attempt to explain their measurable behavior. They are purely intellectual constructs - without empirical content but necessary for scientific reasoning. The cubic electron conceptualization incorporates string theory to assign structure to an ostensibly structureless entity to explain experimental observations and measurements. Mathematical conceptualizations may anticipate aspects of empirical reality later discovered by sophisticated observations and measurement techniques.

The examples cited illustrate that ". . . mathematical as well as logical reasoning make explicit what is implicitly contained in a set of premises while contributing to the content of our knowledge of empirical matters . . . mathematics is entirely indispensable as an instrument for the validation and even for the linguistic expression of such knowledge" [6]. We note that mathematical truths which echo empirical observations and discoveries of experimental science illuminate and often predict them.

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